Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\dfrac{5}{\log_e 5}$ correct to three significant figures. **2**

(b) Simplify $\dfrac{5x}{7} - \dfrac{2x+1}{3}$. **2**

(c) $f(x) = \begin{cases} 3 - 2x & \text{for } x \le 1 \\ x^2 + 2 & \text{for } x > 1 \end{cases}$

Evaluate $f(0) + f(2)$. **2**

(d) Completely factorise $x^3 + 3x^2 - 4x - 12$. **2**

(e) Find the integers a and b such that $\dfrac{1}{1-\sqrt{2}} = a + b\sqrt{2}$ **2**

(f) Solve $|x - 4| = 3$ **2**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate the following functions:

(i) $y = (3x-2)^4$ **1**

(ii) $y = e^{3x-2}$ **1**

(iii) $y = x^2 \cos 2x$ **2**

(iv) $y = \dfrac{x}{\log_e x}$ **2**

(b) Evaluate

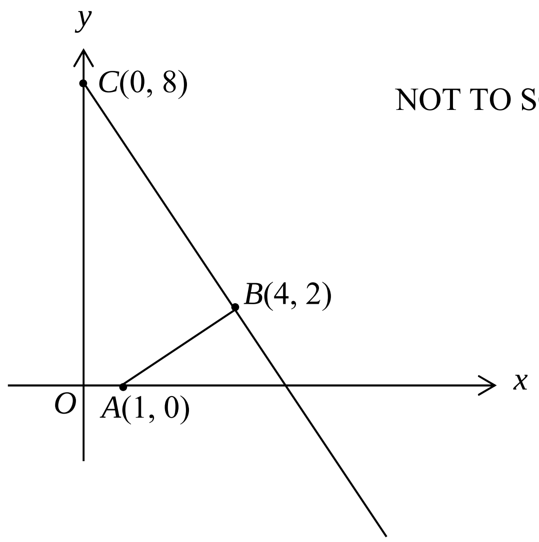
(i) $\displaystyle\int_1^2 \frac{1}{x^3}\,dx$ **2**

(ii) $\int_0^3 e^{-4x}dx$ **2**

(c) Find the equation of the tangent to the curve $y = 2\tan x$ at the point on the curve where $x = \dfrac{\pi}{4}$. **2**

Question 3 (12 marks) Use a SEPARATE writing booklet

(a)



The diagram shows the points $A(1, 0)$, $B(4, 2)$ and $C(0, 8)$ in the Cartesian plane.

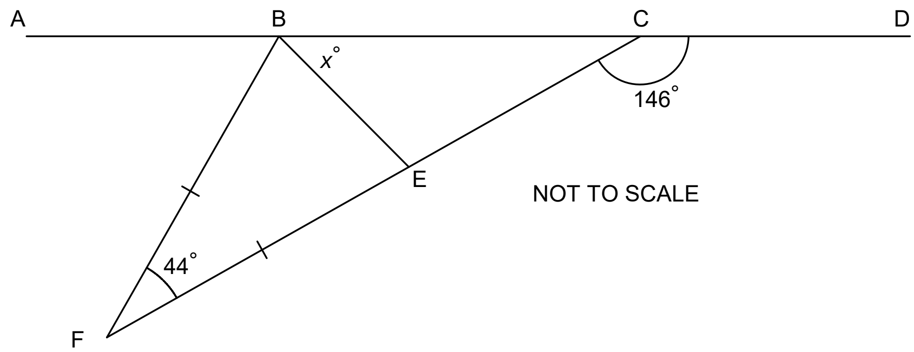
(i) Show that the equation of BC is $3x + 2y - 16 = 0$. **2**

(ii) Show that $\angle ABC$ is $90°$. **2**

(iii) Find the length of AB. **2**

(iv) Find the equation of the circle with centre A that passes through B. **2**

(b)



In the diagram, $ABCD$ is a straight line, and E lies on CF.
$BF = EF$, $\angle BFE = 44°$, $\angle DCE = 146°$, $\angle CBE = x°$.

(i) Find the value of x giving reasons. **3**

(ii) State why $BE = EC$. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) If α and β are the roots of the equation $2x^2 - 5x + 3 = 0$ find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

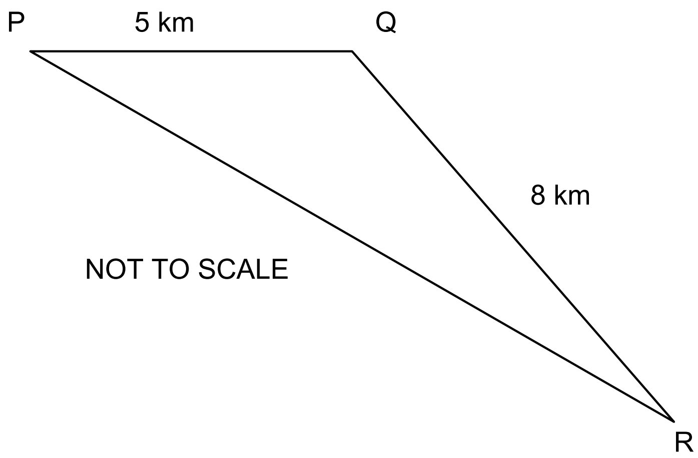
(iii) $\alpha^2\beta + \alpha\beta^2$ **2**

(b) Find the values of k for which the equation $x^2 + 2kx + (3k - 2) = 0$ has real roots. **3**

(c) I walk 5km due east from P to Q, then 8km on a bearing of $130°$ to a point R.

(i) Use the Cosine Rule to find the straight line distance between my starting point and finishing point. **2**

(ii) What is the bearing of P from R? **3**



Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) A function $f(x)$ is defined by

$$f(x) = 4x^3 - x^4$$

(i) Find all solutions of $f(x) = 0$. **2**

(ii) Find the coordinates of any stationary points of the graph of $y = f(x)$ and determine their nature. **4**

(iii) Hence sketch the graph of $y = f(x)$ in the domain $-1 \le x \le 4$, showing the stationary points and points where the curve meets the x-axis. **2**

(b) A researcher is studying the increase of a population of rabbits in the North of the State. He concurs the population is given by the equation $A = 1000e^{0.15t}$ where t is the time in days since the study began.

Find:

(i) the initial population of the rabbits in the study. **1**

(ii) the number of rabbits 7 days into the study. **1**

(iii) on which day the population will have increased to 250 000. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\sum_{r=1}^{\infty} \left(\frac{2}{3}\right)^r$ **3**

(b) An enterprising year twelve student began a paper delivery round in his local neighbourhood. His earnings for the first three months formed a Geometric Progression as shown in the table below.

Month	**1**	**2**	**3**
Earnings ($)	**200**	**300**	**450**

If his earnings continued to increase at the same rate,

(i) how much did he earn in the fourth month ? **1**

(ii) what was his total earnings in the first year of business ? **2**

(iii) how long would it take, to the nearest month, for the student to earn $10000 in total ? **3**

(c) The probabilities that Alex, Bob and Colin will pass the next Probability test are 0.9, 0.8 and 0.7 respectively.

(i) Show that the probability that they all pass is greater than 50%. **1**

(ii) Find the probability that at least one of the three boys pass the test. **2**

Question 7 (12 marks) Use a SEPARATE writing booklet

(a) The displacement of a particle moving in a straight line is given by:

$$x = t^3 - \tfrac{7}{2}t^2 + 2t - 1 \quad \text{(where } t \text{ is in seconds, } x \text{ is in metres).}$$

Find:

(i) the particle's initial displacement. **1**

(ii) the acceleration of the particle after 2 seconds. **3**

(iii) when the particle is at rest. **1**

(iv) the <u>total distance</u> the particle travels between $t = 1$ and $t = 3$ seconds. **2**

(b) The rate of flow of water into a large container is given by:

$$\frac{dV}{dt} = \frac{30}{t+1}$$

where V is in litres and t is in minutes.

Initially, there is 40 litres of water in the container.

(i) Find the volume of water in the container after 4 minutes. **3**

(ii) How long does it take for the container to hold 160 litres? **2**

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $\log(6x-1)-\log(x+2)=\log 4$. **2**

(b) Consider the function defined by $f(x)=e^{x}(1-x)$ for $-3\le x\le 1$.

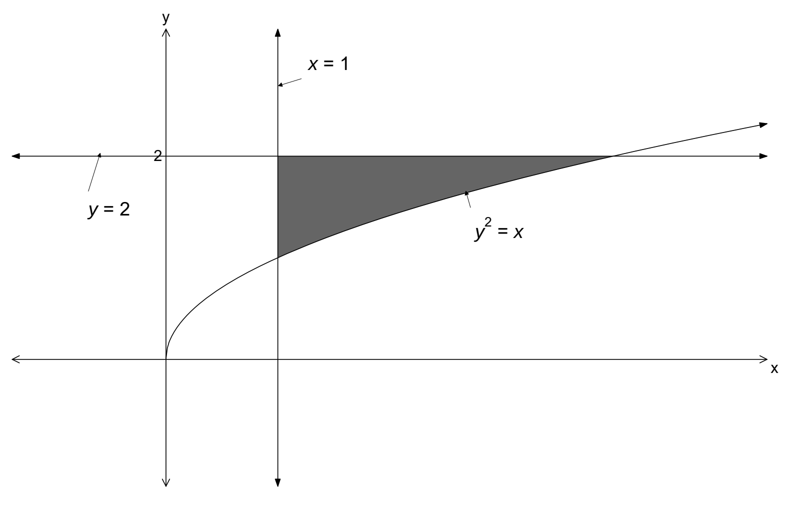
(i) ***Copy and complete the table of values on your answer sheet.*** **2**

Give values correct to 2 decimal places where necessary.

x	-3.00	-2.00	-1.00	0.00	1.00
$f(x)$	0.20				

(ii) Use Simpson's Rule with 5 function values to approximate the area between the curve $f(x)=e^{x}(1-x)$ and the x-axis for $-3\le x\le 1$. **2**

(c)



The shaded region in the diagram is the area bounded by the lines $y=2$ and $x=1$, and the parabola $y^2=x$.

This region is rotated about the y-axis. Find the volume of the solid formed. **4**

(d) Solve the following equation for α (correct to 2 decimal places where necessary).

$$2\cos^2\alpha - 3\sin^2\alpha + 4\sin\alpha = 2$$

for $0°\le\alpha\le 90°$ **2**

Question 9 (12 marks) Use a SEPARATE writing booklet

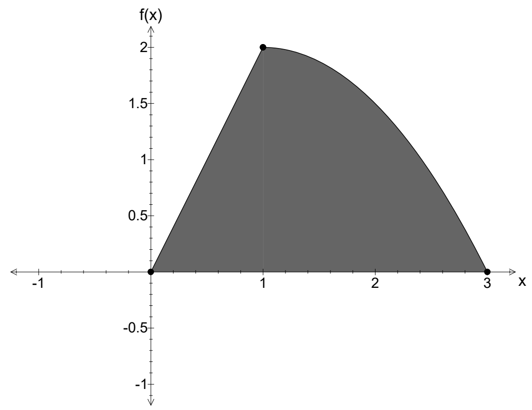
(a) A new golf club is being designed for playing shots under bushes. The shape of its face is shown in the graph below.

Its curved part has the equation

$$y = 2 - \frac{(x-1)^2}{2}, \text{ whilst its}$$

straight parts have the equations

$y = ax + b$ and $y = 0$.



(i) Find the values of the constants a and b. **1**

(ii) Find the area of the face of this new golf club. **4**

(b) Helen borrows \$300 000 at 6% p.a. interest rate. She aims to pay the loan back in equal monthly instalments of \$$M$ over 25 years.

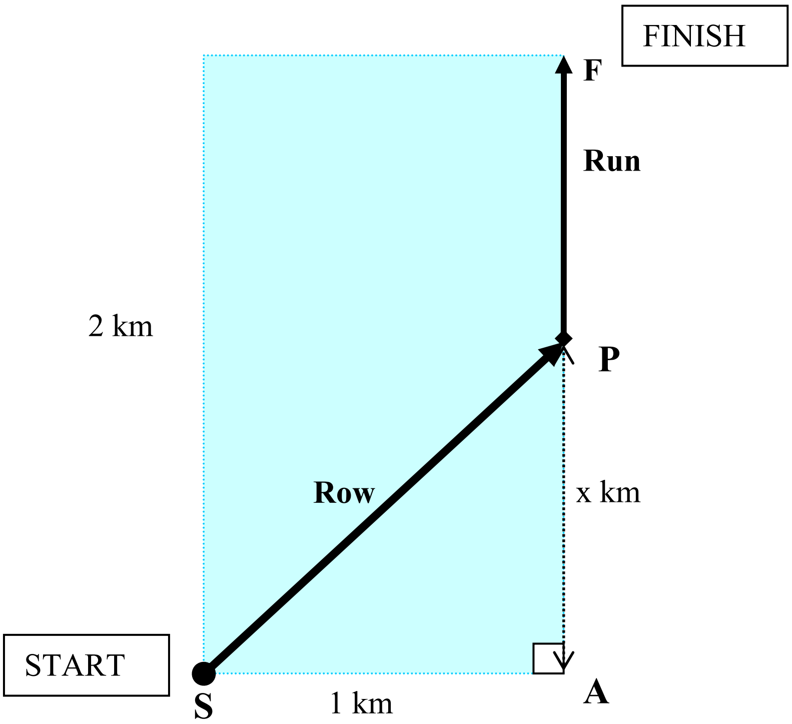
(i) Show that immediately after making her third monthly instalment, Helen owed

$$A_3 = \$[300000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)]$$ **2**

(ii) Calculate the value of M. **2**

(iii) At the end of five years, the interest rate is increased to 7.2% per annum and Helen changes her repayments to \$2600 per month. How many more months are needed to pay off the remainder of the loan? **3**

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**



(a)

The diagram shows a straight section of a rowing course, 1 km wide and 2 km long. Sharon starts at S, rows in a straight line to the opposite bank gets out of the boat then runs to the finish line. Let the distance AP be *x* kilometres.

If Sharon can row at 6 km/hr and run at 10 km/hr,

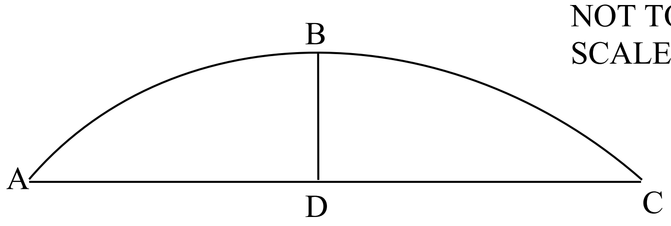
(i) Show that the time T, in hours, that Sharon takes to reach the finish line is given by **3**

$$T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}$$

(ii) Show that if Sharon wishes to minimise the time taken to complete her course, then she should row to a point $\frac{3}{4}$ kilometres from A. **4**

Question 10 (continues) **Marks**

(b)



ABC is a arc of a circle of radius r. If BD was extended the line would pass through the centre of the circle.

Draw the diagram in your answer booklet and indicate the point O which is the centre of the circle.

(i) If $AD = DC = 1$ m and $BD = 10$ cm, show that the radius $r = 505$ cm. **2**

(ii) Find the area of the segment ABCD to the nearest square cm. **3**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\, dx = \ln x\;, \quad x > 0$$

$$\int e^{ax}\, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

YR 12 Trial Maths 2007

Q1 (a) 3.11 ✓✓ (1 for not 3 sig figs)

(b) $\frac{15x}{21} - \frac{7(2x+1)}{21}$ ✓

$= \frac{x-7}{21}$ ✓ (2)

(c) $f(0) + f(2) = 3 + 6$ ✓✓ (1 each)

$= 9$

(d) $x^2(x+3) - 4(x+3)$ ✓

$= (x+3)(x^2-4)$

$= (x+3)(x+2)(x-2)$ ✓ (2)

(e) $\frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ ✓

$= \frac{1+\sqrt{2}}{1-2}$

$= -1-\sqrt{2}$

$a=-1 \quad b=-1$ ✓ (2)

(f) $x-4 = 3$

$x = 7$ ✓

$x-4 = -3$

$x = 1$ ✓ (2)

part (a) issue with understanding of sig figs

(b) usual problem with negative in 2nd ~~bracket~~ term

(c) Some worked out individual values but failed to add!

Some did not understand piece wise function at all

(d) Quite a few had no idea

Some (not 1) students used factor theorem

Many left (x^2-4) as a factor

(e) Many did not realise that $\frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$ (instead $-1+\sqrt{2}$)

(f) Done well.

2 (a)(i) $y=(3x-2)^4$

$\therefore \frac{dy}{dx} = 4(3x-2)^3.3 = 12(3x-2)^3$ ✓

(ii) $y = e^{3x-2}$

$\therefore \frac{dy}{dx} = 3.e^{3x-2}$ ✓

(iii) $y = x^2\cos 2x$

$\therefore \frac{dy}{dx} = 2x.\cos 2x - 2x^2 \sin 2x$ ✓ ✓

(iv) $y = \frac{x}{\ln x}$

$\therefore \frac{dy}{dx} = \frac{\ln x - x.\frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$ ✓ ✓

6

(b)(i) $\int_1^2 \frac{1}{x^3}.dx = \left[\frac{-1}{2x^2}\right]_1^2$ ✓

$= \frac{-1}{8} + \frac{1}{2}$

$= \frac{3}{8}$ ✓

(ii) $\int_0^3 e^{-4x}.dx = \left[-\frac{1}{4}e^{-4x}\right]_0^3$ ✓

$= \frac{-1}{4}(e^{-12} - 1) = 0.249998463$ ✓

4

(c) $y = 2\tan x$ $\therefore \frac{dy}{dx} = 2\sec^2 x$ ✓

where $x = \frac{\pi}{4}$: $y = 2\tan\frac{\pi}{4} = 2$

$\frac{dy}{dx} = 2\sec^2\frac{\pi}{4} = 4$

$y - y_1 = m(x - x_1)$ $\therefore$ $y - 2 = 4(x - \frac{\pi}{4})$ ✓

$\therefore$ $y - 4x - 2 + \pi = 0$

2

QUESTION THREE

(a) (i) $m_{BC} = \dfrac{8-2}{0-4}$

$= -\dfrac{3}{2}$ ✓

$y = -\dfrac{3}{2}x + 8$

$2y = -3x + 16$

$3x + 2y - 16 = 0$ ✓

(ii) $m_{AB} = \dfrac{2-0}{4-1} = \dfrac{2}{3}$ ✓

$m_{AB} \times m_{BC} = \dfrac{2}{3} \times -\dfrac{3}{2} = -1$ ✓

$\therefore AB \perp BC$

(iii) $d_{AB} = \sqrt{2^2 + 3^2}$ ✓

$= \sqrt{13}$ units ✓

(iv) Circle with centre $(1,0)$ and radius $\sqrt{13}$

$(x-1)^2 + y^2 = 13$ ✓ ✓

(b) (i) $\angle FEB = 68^\circ$ (base angles of isosceles Δ equal) ✓

$\therefore \angle BEC = 112^\circ$ (angles on a straight line) ✓

$\therefore x = 146^\circ - 112^\circ$ (exterior angle of a Δ thm.)

$= 34^\circ$ ✓

(ii) $\angle BCE = 34^\circ$ (angles on a straight line)

$\therefore \angle EBC = \angle BCE$

$\therefore BE = EC$ (sides opposite equal angles) ✓

12

QUESTION EIGHT

(a) $\log(6x-1) - \log(x+2) = \log 4$

$\log\left(\dfrac{6x-1}{x+2}\right) = \log 4$ (No marks awarded for this line)

$\therefore \dfrac{6x-1}{x+2} = 4$ ✓

$6x - 1 = 4x + 8$

$2x = 9$

$x = \dfrac{9}{2}$ ✓

(b) (i) $f(x) = e^x(1-x)$

x	-3	-2	-1	0	1
$f(x)$	0.20	0.41	0.74	1	0

✓

(ii) Area $= \displaystyle\int_{-3}^{1} e^x(1-x)\,dx$

$\doteqdot \dfrac{1}{3}\left(0.2 + 4\times0.41 + 2\times0.74 + 4\times1\right)$ ✓ ✓

$= 2.44$ units^2 ✓

(c) $V = \pi\displaystyle\int_1^2 y^4\,dy - \pi$ ✓ ✓ (subtract cylinder with $r=1$ and $h=1$) ✓

$= \pi\left[\dfrac{y^5}{5}\right]_1^2 - \pi$

$= \dfrac{31\pi}{5} - \pi$

$= \dfrac{26\pi}{5}$ units^3 ✓

(d) $\cos^2\theta - 3\sin^2\theta + 4\sin\theta = 1$

$1 - \sin^2\theta - 3\sin^2\theta + 4\sin\theta = 1$

$4\sin\theta - 4\sin^2\theta = 0$

$4\sin\theta(1-\sin\theta) = 0$

$\therefore \sin\theta = 0$ or $\sin\theta = 1$ ✓

$\therefore \theta = 0^\circ, 90^\circ, 180^\circ, 360^\circ$ ✓

12

2u Maths Trial 2007

$$\boxed{\begin{matrix} a, b - 7 \\ c - 5 \end{matrix}}$$

Q4

a) i) $\alpha+\beta = \frac{--5}{2} = \frac{5}{2}$ ✓

ii) $\alpha\beta = 3/2$ ✓

iii) $\alpha\beta(\alpha+\beta)$ ✓ $= 3/2 \times \frac{5}{2}$

$= 15/4$ ✓

b) $\Delta \geq 0$ ✓

$(2k)^2 - 4(1)(3k-2) \geq 0$ ✓

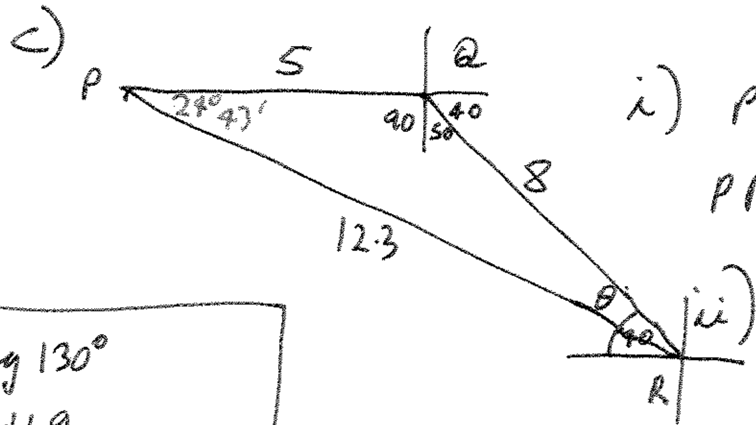
$4k^2 - 12k + 8 \geq 0$

$k^2 - 3k + 2 \geq 0$

$(k-2)(k-1) \geq 0$

$k \geq 2$ or $k \leq 1$ ✓

c)



i) $PR^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 140$ ✓

$PR = 12.3$ ✓ $(12.2590\ldots)$

ii) $\frac{\sin\theta}{5} = \frac{\sin 140}{12.3}$ ✓

$\theta = 15° 12'$ ✓

$15° 08'$ (using 12.3)

$14° 47'$ (using cosine Rule 12.3)

Bearing $= 270 + 40 - 15°12'$

$= 294° 48'$

or $294° 52'$ ✓

or

$360 - 50 - \theta$ ✓

using 130°

$a = 11.9$

$\theta = 18°47'$

Bearing $= 291°33'$

5 ~~b~~. $f(x) = 4x^3 - x^4$

$f'(x) = 12x^2 - 4x^3$

$f''(x) = 24x - 12x^2$

(i) $f(x) = 4x^3 - x^4 = 0$

$x^3(4 - x) = 0$

$x = 0, 4$

(ii) For st pt $f'(x) = 0$

$12x^2 - 4x^3 = 0$

$4x^2(3 - x) = 0$

$x = 0, 3$

when $x = 0$

$f(0) = 0$

$x = 3$

$f(3) = 4(3)^3 - 3^4$

$= 108 - 81$

$= 27$

Test the nature

$f''(x) = 24x - 12x^2$

$f''(0) = 0$

check concavity

x	0^-	0	0^+
$f''(x)$	$-$		$+$

$\therefore$ $(0,0)$ is a horizontal point of inflexion

$f''(x) = 24x - 12x^2$

$= 24(3) - 12(3)^2$

$= -36$

As $f''(x) < 0$ the curve is concave down

$\therefore$ $(3, 27)$ is a local max

NB Must check concavity if using the 2nd derivative

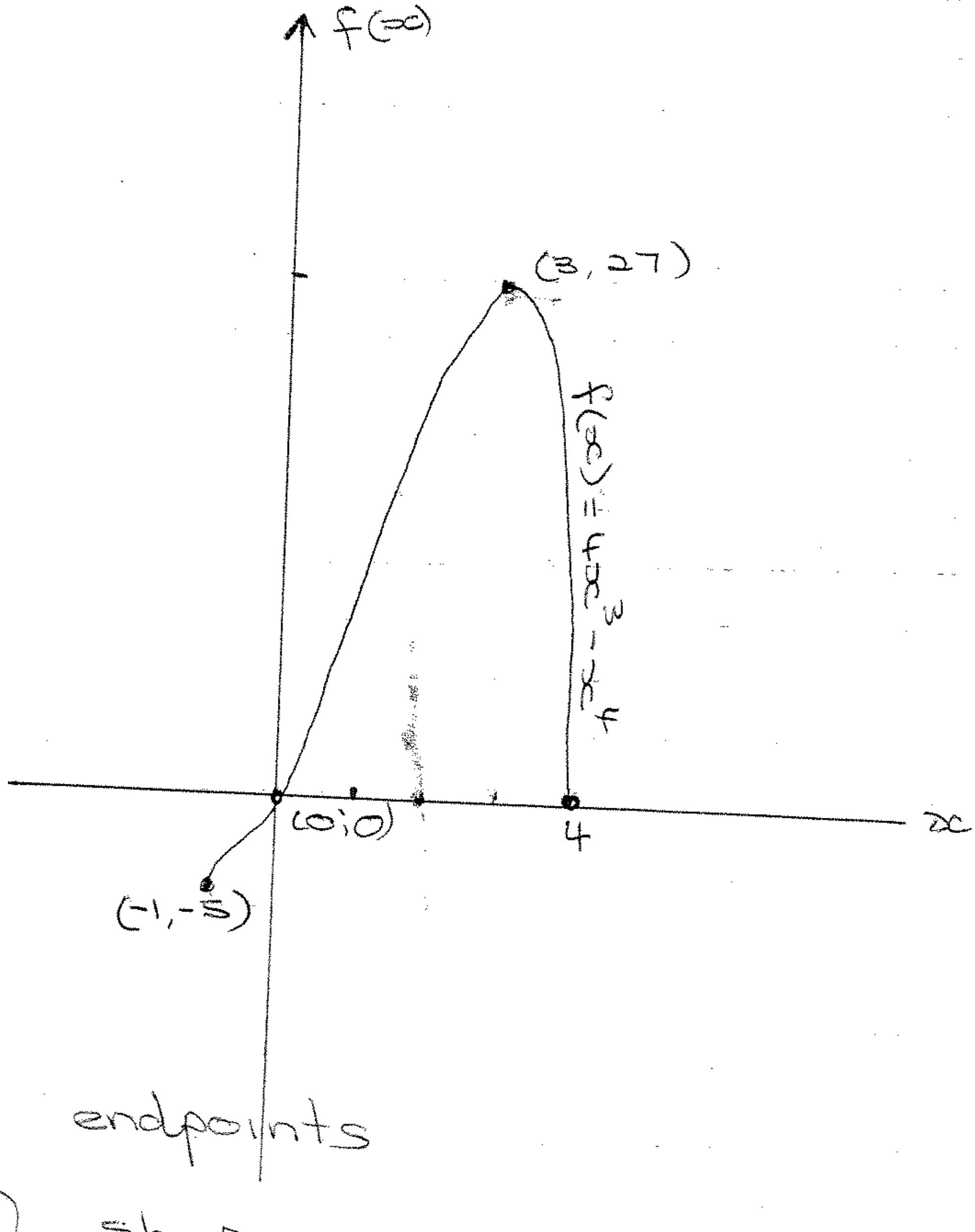
Endpoints

when $x = -1$

$$f(-1) = 4(-1)^3 - (-1)^4$$
$$= -4 - 1$$
$$= -5$$

$x = 4$

$$f(4) = 4(4)^3 - 4^4$$
$$= 0$$



① endpoints

① shape

b) $A = 1000$ |

ii) $A = 1000e^{0.15 \times 7}$

$= 1000e^{1.05}$

$\doteqdot 2857.65118$

$= \begin{cases} 2858 \\ 2857 \end{cases}$ to nearest whole no |

0 marks if they answered as a decimal

(iii) $250000 = 1000e^{0.15t}$

$250 = e^{0.15t}$

$\ln 250 = \ln e^{0.15t}$ |

$0.15t = \ln 250$

$t = \dfrac{\ln 250}{0.15}$

$\doteqdot 36.8097394$

In the 37 day |

Question 6

a,b	9
c	3

a) $S_\infty = \frac{a}{1-r}$ ✓

$a = 2/3 \quad r = 2/3$ ✓

$S_\infty = \frac{2/3}{1-2/3} = \frac{2}{3} \times \frac{3}{1}$ ✓

$= 2$

b) i) $450 \times 1\frac{1}{2} = 675$ ✓

ii) $S_{12} = \frac{200(1.5^{12}-1)}{1.5-1}$ ✓

$= \$5149.54$ ✓

iii) $70000 = \frac{200(1.5^n-1)}{1.5-1}$ ✓

$\frac{35000}{200}$ $\cancel{140000} = 200(1.5^n-1)$

$\cancel{\frac{140000}{200}}$

$175 \ \cancel{200} = 1.5^n - 1$ ✓

$\log 1.5^n = \log \cancel{701}\ 176$

$n = \frac{\log \cancel{175}}{\log 1.5}$

$= \cancel{16.16}\ 12.737$ ✓

13 months

c) i) $P(PPP) = 0.9 \times 0.8 \times 0.7 = 0.504 > 50\%$ ✓

ii) $P(\text{atleast 1 pass}) = 1 - P(\text{all fail})$ ✓

$= 1 - (0.1 \times 0.2 \times 0.3)$

$= 0.994$ ✓

7. $x = t^3 - \frac{7}{2}t^2 + 2t - 1$

$\dot{x} = 3t^2 - 7t + 2$

$\ddot{x} = 6t - 7$

(i) initial displacement -1m or one metre to the left ①

(ii) $\dot{x} = 3t^2 - 7t + 2$ ①

$\ddot{x} = 6t - 7$ ① at $t = 2$

$= 6(2) - 7$

$= 5$ ①

acceleration is $5\,ms^{-2}$

(iii) $3t^2 - 7t + 2 = 0$ at rest $\dot{x} = 0$

$(3t - 1)(t - 2) = 0$

$t = \frac{1}{3}, 2$ ① both must be correct

(iv) At $t = 1$, $x = 1^3 - \frac{7}{2}(1)^2 + 2(1) - 1$

$= -1\frac{1}{2}$

At $t = 2$ $x = 2^3 - \frac{7}{2}(2)^2 + 2(2) - 1$

$= -3$

① mark for showing understanding of concept

At $t = 3$ $x = 3^3 - \frac{7}{2}(3)^2 + 2(3) - 1$

$= \frac{1}{2}$

Total distance $= 1\frac{1}{2} + 3\frac{1}{2}$

$= 5$

① correct answer

NB 0 marks if students did at $t = 1$, $x = -1\frac{1}{2}$ at $t = 3$, $x = \frac{1}{2}$ total distance $= 2$

2 marks if correct integration was used

b) i) $\frac{dv}{dt} = \frac{30}{t+1}$

$V = \int \frac{30}{t+1} \, dt$

$= 30 \log_e (t+1) + c$ 1 at $t = 0$ $V = 40$

$40 = 30 \log_e (t+1) + c$

$\therefore c = 40$ 1

$V = 30 \log_e (t+1) + 40$ at $t = 4$

$= 30 \log_e (4+1) + 40$

$\doteqdot 88.283187$ No penalty for rounding

$\therefore$ Volume is 88L to nearest L 1

ii) $160 = 30 \log_e (t+1) + 40$

$120 = 30 \log_e (t+1)$

$4 = \log_e (t+1)$ 1

$t+1 = e^4$

$t = e^4 - 1$

$\doteqdot 53.59815003$ No penalty for rounding 1

$= 54$ to nearest whole no

time is 54min to nearest min

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Q9 (a) (i) $a = 2 \quad b = 0$

(ii) $\text{Area} = \frac{1}{2} \times 1 \times 2 + \int_1^3 2 - \frac{(x-1)^2}{2}\, dx$

$= 1 + \left[2x - \frac{(x-1)^3}{6} \right]_1^3$

$= 1 + \left[\left(6 - \frac{8}{6}\right) - 2 \right]$

$= 3\frac{2}{3} \ u^2$ (4)

(b) (i) If A_i is amount owed after i^{th} instalment.

$A_1 = 300000 \times 1.005 - M$ monthly interest rate 0.5%

$A_2 = (300\,000 \times 1.005 - M) 1.005 - M$

$= 300000 \times 1.005^2 - M \times 1.005 - M$

$A_3 = (300\,000 \times 1.005^2 - M \times 1.005 - M) 1.005 - M$

$= 300\,000 \times 1.005^3 - M \times 1.005^2 - M \times 1.005 - M$

$= 300\,000 \times 1.005^3 - M\,(1 + 1.005 + 1.005^2)$ (2)

(ii) $A_{300} = 300\,000 \times 1.005^{300} - M\,(1 + 1.005 + \ldots + 1.005^{299})$

Loan paid when $A_{300} = 0$

$0 = 300\,000 \times 1.005^{300} - \frac{M \times 1\,(1.005^{300} - 1)}{1.005 - 1}$

$M = \frac{300\,000 \times 1.005^{300} \times 0.005}{(1.005^{300} - 1)}$

$= \$1932.90$ (2)

(iii) After 5 years amount owed is $A_{60} = 300000 \times 1.005^{60} - \frac{1932.9 \times (1.005^{60} - 1)}{0.005}$

$= \$269796.55$

Find n such that $0 = 269796.55 \times 1.006^n - \frac{2600 \times (1.006^n - 1)}{0.006}$

$0 = 0.6226\ldots \times 1.006^n - 1.006^n + 1$

$1.006^n = 2.64976\ldots$

$n \log 1.006 = \log 2.64976\ldots$

$n = \frac{\log 2.64976\ldots}{\log 1.006}$

$= 163$ months (3)

Y12 Trial Maths 2007

Q9 Comments

(a)(i) Many students did far too much work here (e.g using simultaneous equations)

(ii) Most used triangle to find first area
Some horrible integration.
Some "multiplied through by 2" to remove fractions (?).
A couple failed to add areas.

(b)(i) Many had no idea
Some just wrote out A_1 and A_2 in a similar way to A_3 with no explanation (only 1 mark could be granted).
Best solutions involved iterative process.

(ii) Some failed to convert months.
As level of accuracy not stated, marks were not deducted for rounding issues.
Some put 301 or 299 in sum of series

10. (a) $SP = \sqrt{1+x^2}$

$PF = 2-x$

$T_{row} = \dfrac{SP}{6} = \dfrac{\sqrt{x^2+1}}{6}$

$T_{run} = \dfrac{PF}{10} = \dfrac{2-x}{10}$

$\therefore T = \dfrac{\sqrt{x^2+1}}{6} + \dfrac{2-x}{10}$

(b) $$\frac{dT}{dx} = \frac{\frac{1}{2}(x^2+1)^{-1/2}.2x}{6} - \frac{1}{10}$$

$$= \frac{2x}{12\sqrt{x^2+1}} - \frac{1}{10}$$

$$= \frac{x}{6\sqrt{x^2+1}} - \frac{1}{10}$$

$$= \frac{5x - 3\sqrt{x^2+1}}{30\sqrt{x^2+1}}$$

$\therefore \dfrac{dT}{dx} = 0$ when $5x - 3\sqrt{x^2+1} = 0$

$$5x = 3\sqrt{x^2+1}$$
$$25x^2 = 9(x^2+1)$$
$$25x^2 = 9x^2+9$$
$$16x^2 = 9$$
$$x^2 = 9/16$$
$$x = \pm 3/4$$

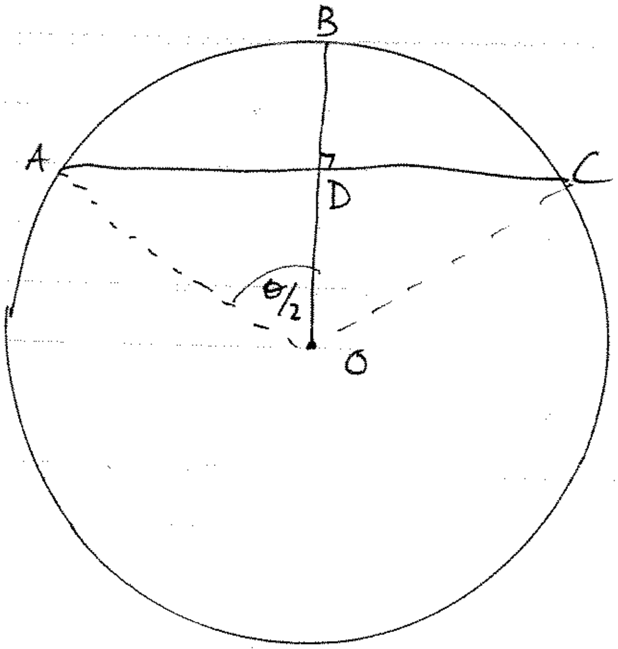
$\therefore$ stat. pt. at $x = 3/4$ (ignore neg. length)

$$\frac{d^2T}{dx^2} = \frac{6\sqrt{x^2+1}\,.\,1 - x.\,6.\,\frac{1}{2}(x^2+1)^{-1/2}.\,2x}{36(x^2+1)}$$

$$= \frac{6\sqrt{x^2+1} - 6x^2(x^2+1)^{-1/2}}{36(x^2+1)}$$

$\therefore$ at $x = \frac{3}{4}$ $\dfrac{d^2T}{dx^2} = \dfrac{6\sqrt{\frac{25}{16}} - 6.\frac{9}{16}.\frac{1}{\sqrt{25/16}}}{36.\frac{25}{16}} = \dfrac{\frac{30}{4} - \frac{216}{80}}{225/4} = \dfrac{32}{88}$

(b)



(i) $AD = 100 \qquad BD = 10$

$\therefore OD = r - 10$

$\therefore (r-10)^2 + 100^2 = r^2$

$\therefore r^2 - 20r + 100 + 10000 = r^2$

$\therefore -20r + 10100 = 0$

$\therefore 20r = 10100$

$\therefore r = 505$ cm

(ii) $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$\sin\frac{\theta}{2} = \frac{100}{505} \quad \therefore \frac{\theta}{2} = \sin^{-1}\frac{100}{505}$

$\therefore \theta = 2\sin^{-1}\frac{100}{505}$

$= 0.39867461$

$\therefore A = \frac{1}{2} . 505^2 . \left[2\sin^{-1}\frac{100}{505} - \sin\left(2\sin^{-1}\frac{100}{505}\right)\right]$

$= 1335.996203$

$= 1336 \text{ cm}^2$